Astroparticle Physics, NAASPH

Ad van den Berg & Olaf Scholten
KVI - Center for Advanced Radiation Technology NL-9747 AA Groningen

Exam; 31 March 2015; 09:00 -12:00; 5419.0013
5 problems (total of 57 points).
Write the solution of every problem on a separate piece of paper with name and student number.

Write clearly, it should be readable.

Problem 1 (10 pnts in total)
Cherenkov emission is an important technique used to detect charged particles.

1 pnts a. Explain why this technique can only be used for the detection of charged particles.

1 pnts b. The emission through Cherenkov radiation will only occur when the velocity of a particle is larger than or equal to the velocity of light in the medium. The relevant equation is:

$$\beta = \frac{v}{c} \geq \frac{1}{n}$$

What is the physics meaning of the parameter n?

2 pnts c. The direction of the emitted light is given by the Cherenkov angle: θ_C. Make a sketch of the direction of the particle and the direction of the emitted light; indicate the Cherenkov angle in this picture.

2 pnts d. The value of the Cherenkov angle is given as:

$$\theta_C = \arccos \frac{1}{n\beta}$$

Use the relations $p = \gamma m_0 \beta c$ and $E^2 = p^2c^2 + m_0^2c^4$ to obtain an expression for m_0 in terms of p, n, and θ_C.
Here p is momentum, γ is the Lorentz factor ($\gamma = 1/\sqrt{(1-\beta^2)}$), m_0 the rest mass of a particle, and E is the total energy of a particle (sum of rest energy and kinetic energy).

2 pnts e. Fluorescence emission is based on a completely different physics principle as Cherenkov emission. Describe the emission mechanism of fluorescence emission.

2 pnts f. Name two differences (apart from the emission mechanism) between Cherenkov and fluorescence emission.

Problem 2 (15 pnts in total)

1 pnts a.Magnetic fields play a very important role in the acceleration of charged particles. However, static magnetic fields cannot accelerate charged particles. Explain why.

3 pnts b. Describe the basic features of the following acceleration mechanism: a) cyclotron mechanism, b) 1st order Fermi acceleration, c) 2nd order Fermi acceleration.

3 pnts c. Give for each of these acceleration scenarios the possible location or site where they are likely to occur.

2 pnts d. What is the difference between synchrotron radiation and bremsstrahlung.

1 pnts e. The power P radiated by a relativistic electron of energy E in a transverse magnetic field B through synchrotron radiation can be worked out through classical electrodynamics as:

$$P = \frac{e^2c^3}{2\pi}C_\gamma E^2 B^2,$$

where e is the charge of the electron, c the velocity of light, and C_γ is a numerical constant (9×10^{-5} m GeV^{-3}).
An electron orbits around a pulsar at a radial distance given as r. Derive an expression for the radius r in terms of B and E.

2 pnts f. Assume that r is 1000 km and the energy E is given as 1 TeV, how much is the energy loss of the electron for each turn around the pulsar.

3 pnts g. Pulsars are rapidly rotating neutron stars. Describe a mechanism which might accelerate charged particles near a pulsar.

Problem 3 (10 pnts in total)
Dark Matter has not yet been proven to exist. However, there are many indications that the energy density in the Universe today should be attributed for more than 20% to Dark Matter (DM). For our own galaxy the rotation velocity of the stars is approximately equal to 220 km/s from a radius of 1 kpc up to 15 kpc.

4 pnts a. Describe the measurements on the basis of which Oort could deduce the velocity of the stars in the vicinity of our Sun.

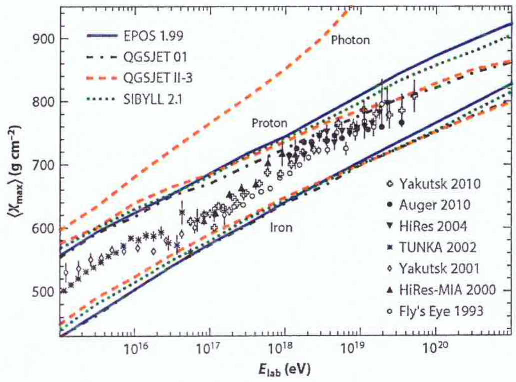
3 pnts b.Assume that the DM is in the form of a spherical halo. Use the constancy of the rotational velocity to derive the radial dependence of the Dark Matter density $\rho_\chi(r)$.

1 pnts c. What is the local density of dark matter near the position of the Sun?

2 pnts d. The density you obtain in the way described in this problem is about a factor two larger that the true DM density near the Sun ($\tilde{0}.3$ GeV/cm^3). Explain the difference.

Problem 4 (12 pnts in total)

When high-energy cosmic rays enter the atmosphere of the Earth they create a cascade of secondary particles, called an extensive air shower. At an energy of 10^{16} eV a photon penetrates much deeper in the atmosphere than a proton which in turn penetrates deeper than an Iron nucleus (see figure).



2 pnts a. Give two reasons why an iron-induced shower has a smaller value for X_{max} than a proton-induced shower at the same energy.

2 pnts b. Give the reason why a proton induced shower has a smaller value for X_{max} than a photon-induced shower at the same energy.

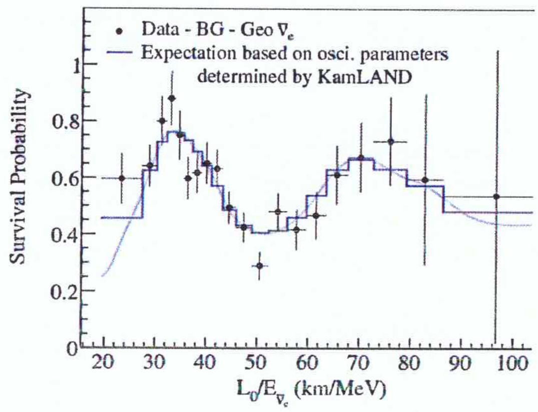
4 pnts c. By how much, using the Heitler model, do you expect X_{max} to increase for a photon-induced shower when the energy increases by a factor 10? Compare this to the results given in the figure.

2 pnts d. Indicate the general structure of the shower profile (the number of particles as function of height) and give the reason why this dependence as function of height is different for neutrinos as well as muons.

2 pnts e. Give the reason why a photon-induced shower does have a strong electro-magnetic component and hardly any energy in the hadronic component while a proton-induced shower also has a dominant electromagnetic component.

Problem 5 (10 pnts in total)

At KamLAND neutrino oscillations have been measured using neutrinos produced in various reactors in Japan. The results of the measurements are displayed in the figure.



2 pnts a. Explain in less than three lines the essence of neutrino oscillations.

4 pnts b. Use the data in the figure to estimate Δm^2.

2 pnts c. Explain why the amplitude of the oscillations in the figure decrease for large values of L/E.

Some numbers

Electron mass $m_e c^2 = 511\,\mathrm{keV}$; Muon mass $m_\mu c^2 = 106\,\mathrm{MeV}$; Pion mass $m_\pi c^2 = 140\,\mathrm{MeV}$;
Proton mass: $m_p c^2 = 0.938\,\mathrm{GeV}$
Conversion: $1\,\mathrm{eV/c^2} = 1.78 \times 10^{-36}\,\mathrm{kg}$
Boltzmann's constant: $k = 8.62 \times 10^{-11}\,\mathrm{MeV/K}$
Planck's constant: $h = 4.1 \times 10^{-15}\,\mathrm{eV\ s}$ and $\hbar c = 2 \times 10^{-7}\,\mathrm{eV\,m}$
Weak coupling constant: $G_F = 10^{-5}\,\mathrm{GeV^{-2}}$
Avogadro's number: $N_A = 6 \times 10^{23}\,/\mathrm{mol}$
Solar Mass: $\mathrm{M}_\odot = 1.99 \times 10^{30}\,\mathrm{kg}$
Parsec: $1\,\mathrm{pc} = 3.1 \times 10^{16}\,\mathrm{m}$
Velocity of Sun w.r.t. center Milkyway: $V_\odot = 220\,\mathrm{km/s}$
Velocity of Earth in orbit around the Sun: $V_\oplus = 30\,\mathrm{km/s}$
Typical galactic dark matter density: $\rho_{DM} = 9\ \mathrm{k\ M_\odot/pc^3}$

Neutrino-oscillation physics

$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$
$\Delta m^2_{sol} \approx 0.8 \times 10^{-4}\ \mathrm{eV}^2$; $\Delta m^2_{atm} \approx 2.5 \times 10^{-3}\ \mathrm{eV}^2$
MSW: $\Delta m^2_m = B\Delta m^2$; $\sin 2\theta_m = \frac{\sin 2\theta}{B}$; $A = 2\sqrt{2} G_F N_e E/\Delta m^2$; $B^2 = (\cos 2\theta - A)^2 + \sin^2 2\theta$

Air-shower physics

At 10 km height the density of the atmosphere is $0.4 \times 10^{-3}\,\mathrm{g\,cm^{-3}}$.
$X_0 = 37\,\mathrm{g\,cm^{-2}}$; the penetration depth for pions in air is $\lambda_\pi = 120\,\mathrm{g\,cm^{-2}}$, for protons $\lambda_p = 90\,\mathrm{g\,cm^{-2}}$, and for iron is $\lambda_{Fe} = 5\,\mathrm{g\,cm^{-2}}$.
The mean travel distance in vacuum of a pion with energy E is $d_{\pi^0} = \gamma 25 \times 10^{-9}\,\mathrm{m}$ and $d_{\pi^\pm} = \gamma 7.8\,\mathrm{m}$ where the relativistic γ-factor is given by $\gamma = E/mc^2$ and $m_\pi c^2 = 140\,\mathrm{MeV}$.

Integrals

For $c > 0$ we have:

$$\int_0^\infty c\,e^{-cx} dx = 1\ ; \quad \int_0^\infty c\,x\,e^{-cx} dx = 1/c\ ; \quad \int_0^\infty c\,x^2 e^{-cx} dx = 2/c^2$$